Soda-Bottle Water Rockets

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It's hard to beat the fun of launching water rockets on a warm summer day. Of course we are not the first physics teachers to discover this basic fact.[1,2] To be sure, launching small toy-store rockets may lack sufficient excitement, since they rise only five to ten meters into the air and produce a very modest amount of back spray. But the concept is so attractive that we designed a well-controlled, inexpensive, and rather high-performance method for launching 2-ℓ plastic soda bottles that rise 20 or more meters into the air and spray about one liter of water.

In the process of studying these rockets we discovered that their launch can be modeled by a wonderful physics problem that illustrates Newton's second law, the concepts of momentum and relative velocity, fluid flow using Bernoulli's principle and the equation of continuity, and the adiabatic expansion of an ideal gas. In addition, the resulting equations of motion for the rocket, like most "real-world" problems, cannot be solved analytically and therefore require a numerical solution. In this case we used a spreadsheet program. The predictions of our model seemed unbelievable. The rocket should reach speeds well in excess of 150 mph in less than the first two meters, which it covers in one-tenth of a second! Accelerations would be in excess of 100 g's! We were so amazed by these numbers that we were motivated to check their validity experimentally.

We have tested the predictions of our launch theory by studying the motion of the rockets with two methods. First, we actually photographed a launch with a high-speed camera. This method proved effective but too difficult and expensive to use for a systematic study, so we switched to using a "Smart Pulley" to collect our data.

In this paper we first discuss the construction of the 2-ℓ plastic soda-bottle rockets and then present our theory of their motion during launch. Finally, we compare the predictions of our model with actual experimental data.

Construction

A device to launch 2-ℓ plastic soda bottles can be easily and inexpensively built. The basic design consists of two components. The first part is the plumbing to seal the bottle while still allowing air inside to pressurize it. Figure 1 shows both an exploded view and a completely assembled unit. To make the gasket and to get a valve stem, ask for an old inner tube at your local bike shop. A well-stocked hardware store will be able to supply the remaining parts: two half-inch PVC nipples (four to six inches long), a half-inch PVC threaded elbow, and a drip irrigation dripper (Rainbird #PC8100 or equivalent). The final piece of plumbing is a CPVC half-inch pipe to half-inch slip connector (Genova #50305 or equivalent), which is threaded on one side and has a hole for the valve stem on the other. If this part is hard to find, you can just use a half-inch PVC threaded end cap and drill a hole in it for the valve stem. You will also need a little bit of PVC glue and perhaps some Teflon tape for the threads.

Cut off one end of one of the nipples. The length of this nipple is not critical, anywhere around 4 in is fine. Ream out the cut-off end of the nipple until the dripper fits snugly in the hole, glue it in place with PVC glue, and let it thoroughly dry. Cut the gasket out of the old inner tube. Its diameter should be 1.25 in, with a 3/32-in hole in the center. Cut the valve stem from the tube so that it fits snugly through the hole in the end cap. Thread all the pieces together and place the gasket over the dripper to complete the plumbing.

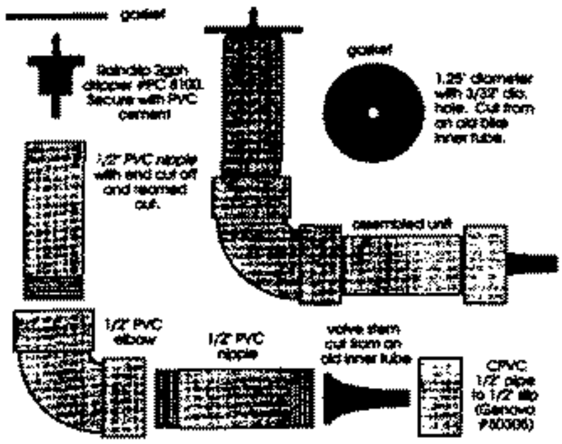

Fig. 1. Exploded and assembled views of plumbing unit.

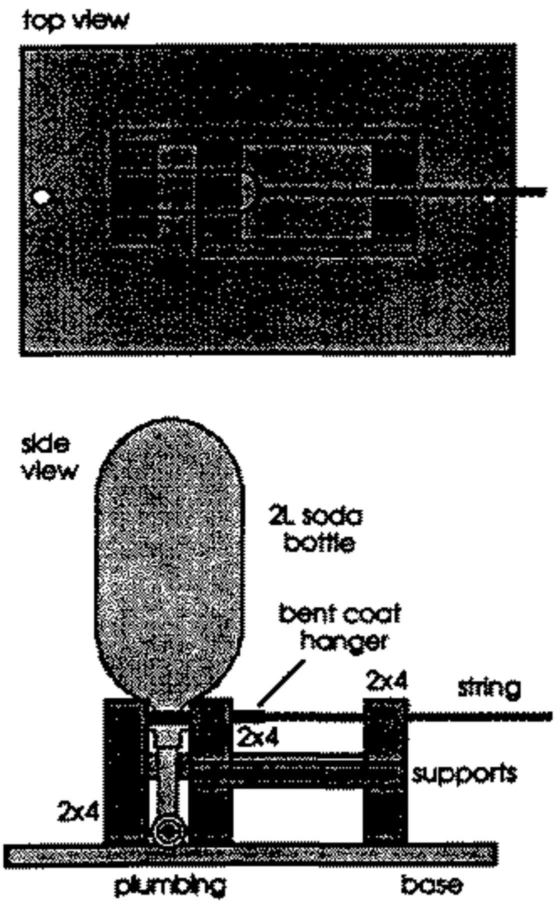

Fig. 2. Top and side views of a basic launch pad.

The second component is the launch pad that holds the bottle in place while it is pressurized and releases the rocket when you are ready. The cheapest and simplest version is shown in Fig. 2. It is built from scrap wood, a bent coat hanger, and a piece of string. You will need a drill to make the necessary holes in the 2 × 4's. The height of the 2 × 4's and the location of the guide holes for the coat hanger depend on the height of the plumbing assembly. We screwed the pieces of wood together for added strength. It is important that the 2 × 4's on either side of the bottle create a tight fit. Also, the coat hanger should make a tight fit just above the heavy plastic ring on the neck of the bottle. This ring rests against the coat hanger as the rocket is pressurized. The third 2 × 4 stops the coat hanger from flying toward you after launch. The holes in the base are for fastening it to the ground with stakes.

We found that it was easiest to start by removing everything from the launch pad and attaching the plumbing to a bike pump. Then we turned the plumbing over and inserted it into the upright soda bottle that was about half-filled with water. Next, we raised or lowered the plumbing until the gasket sealed. Then we flipped the whole apparatus over, placed it into the launch pad, slid the coat hanger over the plastic ring, and pumped up the pressure. A sturdy tug on the string starts the fun.

Safety is extremely important. We have heard anecdotal reports of serious injury caused by premature launch of these

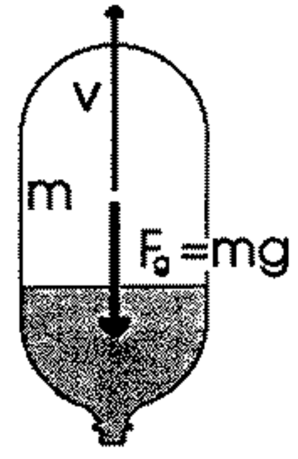

Fig. 3. Water rocket just before Δm is ejected. The mass of the system is m, and it has a velocity v. The net force on the system is just its weight.

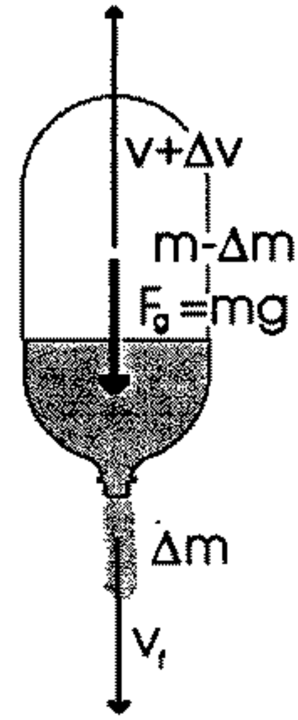

Fig. 4. Water rocket just after Δm is ejected. The ejected water has a mass Δm and a speed v_f. The mass of the rocket changes by Δm and its speed changes by Δv.

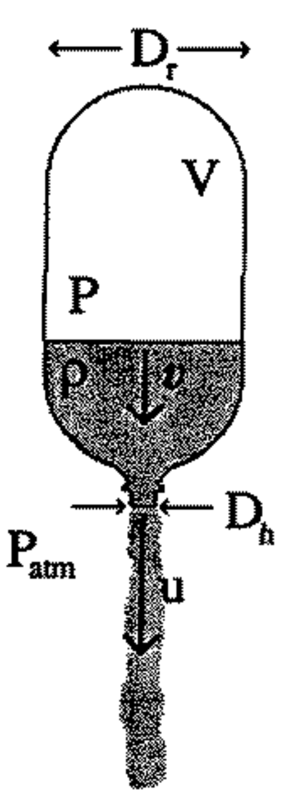

Fig. 5. Water flows out of the rocket. V—volume of air in the rocket; P—pressure inside; D_r—diameter of the rocket; D_h—diameter of the hole; ρ—density of the water; v—speed of the water inside the rocket; u—speed of the water outside the rocket; P_{atm} is the atmospheric pressure.

water rockets. *We suspect* that this is the reason that a commercially available launcher is not on the market.[3] These soda bottles are designed to minimize lawsuits against soft-drink manufacturers, so they seem to present little danger of exploding. We have used the same bottle many tens of times at gauge pressures from 50 to 120 psi[4] without the bottle rupturing. We don't know what happens at higher pressures and we discourage *you from attempting to* find out. During their descent the bottles aren't worth much concern because they are light and gently fall down toward Earth. They are a serious threat only while they are pressurized before launch and as they gain speed, but still have some mass right after launch. To avoid trouble before launch, be sure your launch pad is sturdy enough to withstand the pressure. Stake the launch pad into the ground so that it can't tip over when you pull the string. Stand away from the launch pad when you pump up the pressure and be sure nobody is anywhere near the pad when you launch the rocket. We recommend that you do not leave students unattended while using these water rockets.

We guarantee that you will be thrilled as you try to watch your first launch. The bottle will simply vanish from the pad, water will rain down from great heights, and you will, with some effort, finally spot the rocket between 20 and 30 meters above your head gently drifting downward. As it slowly falls back to Earth, you'll be in a hurry to try it again.

Theory

The motion of a water rocket during launch can be understood by applying Newton's second law, $\Sigma F = \frac{\Delta p}{\Delta t}$. Figure 3 *shows the rocket just before it ejects some water.*[5] *Its initial* momentum is $p_i = mv$, where v is its initial velocity and m is the total mass of the bottle and water. Figure 4 shows the system just after it ejects a small mass of water, Δm, at a speed, v_f. The velocity of the bottle has changed by Δv, so the system now has a final momentum of

$$p_f = (m - \Delta m)(v + \Delta v) - \Delta m\, v_f$$

Since the only force on the system is the weight of the bottle and water, the second law requires

$$-mg = \frac{(m - \Delta m)(v + \Delta v) - (\Delta m)\, v_f - mv}{\Delta t}$$

Doing some algebra, and neglecting the $\Delta m \Delta v$ term because it is small, yields

$$-mg = \frac{m\Delta v - (\Delta m)(v + v_f)}{\Delta t}$$

Solving for the change in velocity of the rocket, Δv, gives

$$\Delta v = -g\Delta t + u\frac{\Delta m}{m}$$

where u is the relative velocity of the ejected water with respect to the rocket, $u = v + v_f$. Note that $\Delta m = \rho \Delta V$ and $m = \rho V_w + m_r$, where ρ is the density of the water, ΔV is the change in volume of the air in the rocket (or equivalently the change in volume of the water in the rocket), V_w is the volume *of water remaining in the rocket, and m_r is the mass of the* empty rocket. Using these quantities,

$$\Delta v = -g\Delta t + u\left(\frac{\rho \Delta V}{\rho V_w + m_r}\right) \qquad (1)$$

The velocity of the rocket can be found once the velocity of the ejected water with respect to the rocket is known. The simplest model of the fluid flow in the rocket treats the rocket

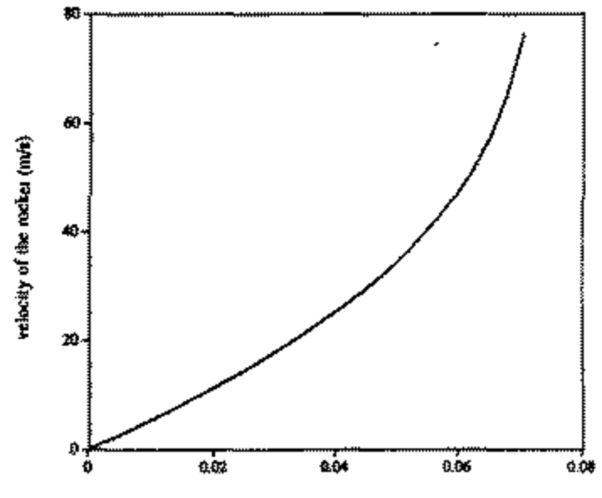

Fig. 6. Velocity of the rocket vs time (m = 48.4 g, D_r = 11.0 cm, D_h = 2.15 cm, V_r = 2.06 ℓ; initial volume of water is 0.70 ℓ; initial gauge pressure is 80 psi).

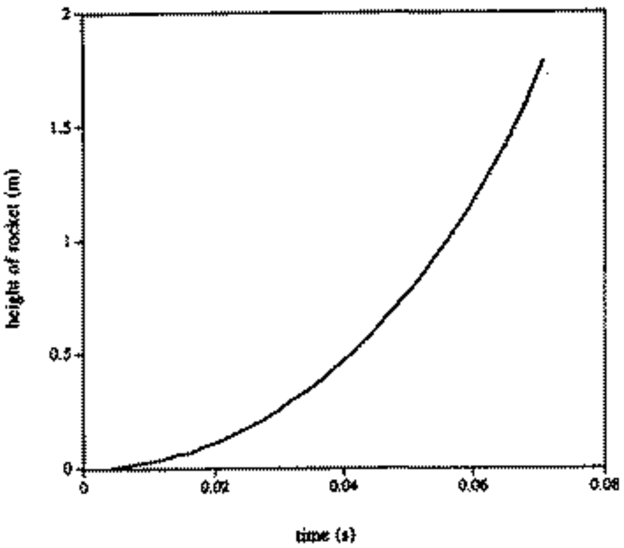

Fig. 7. Height of the rocket vs time (m = 48.4 g, D_r = 11.0 cm, D_h = 2.15 cm, V_r = 2.08 ℓ, the initial volume of water is 0.70 ℓ, and the initial gauge pressure is 80 psi).

like a pipe with water flowing through it. This flow is then described by Bernoulli's equation,[6]

$$\frac{1}{2}\rho \upsilon^2 + P = \frac{1}{2}\rho u^2 + P_{atm}$$

The quantities in this equation are shown in Fig. 5. P is the absolute air pressure in the rocket, V is the volume of air in the rocket, υ is the velocity of the water in the rocket, D_r is the diameter of the rocket, ρ is the density of the water, D_h is the diameter of the hole through which the water escapes, P_{atm} is the atmospheric pressure, and finally, u is the quantity we seek, the velocity at which the water is ejected.

Both u and υ are related to the rate at which the water escapes or equivalently the rate at which the volume of the air increases. This relationship is given by the equation of continuity,[7]

$$\frac{\Delta V}{\Delta t} = \frac{1}{4} u \pi D_h^2 = \frac{1}{4} \upsilon \pi D_r^2 \qquad (2)$$

Solving for υ in terms of u and substituting into Bernoulli's equation yields an expression for the velocity of the ejected water,

$$u = \sqrt{\frac{2(P - P_{atm})}{\rho\left\{1 - \left(\frac{D_h}{D_r}\right)^4\right\}}}$$

The pressure in the rocket during launch varies with the volume of the air inside. The bottle, when it returns to the ground, often contains a fog from condensation of water vapor. This implies that the air inside must have cooled as it expanded. In addition, our results indicate that the gas expands in a very short time, which leads us to suspect that heat has no time to flow into the rocket. We therefore assume that we are dealing with the adiabatic expansion of an ideal gas,[8]

$$P = P_o\left(\frac{V_o}{V}\right)^{\gamma}$$

where P_o is the initial absolute pressure in the rocket and V_o is the initial volume of air in the rocket. For air, which can be considered a diatomic gas, $\gamma = 1.4$.[9] The velocity of the ejected water can now be written as a function of the volume of air remaining inside the rocket,

$$u = u_c\sqrt{\frac{\left(\frac{V_o}{V}\right)^{\gamma} - \frac{P_{atm}}{P_o}}{1 - \left(\frac{D_h}{D_r}\right)^4}} \qquad (3)$$

where u_c is the characteristic velocity given by

$$u_c = \sqrt{\frac{2P_o}{\rho}}$$

The velocity of the rocket, v, as a function of time, t, can be found by the following numerical steps. We assume that $v = 0$ at $t = 0$.

1. Find values for:

P_o—the initial absolute air pressure in the rocket (<135 psi).

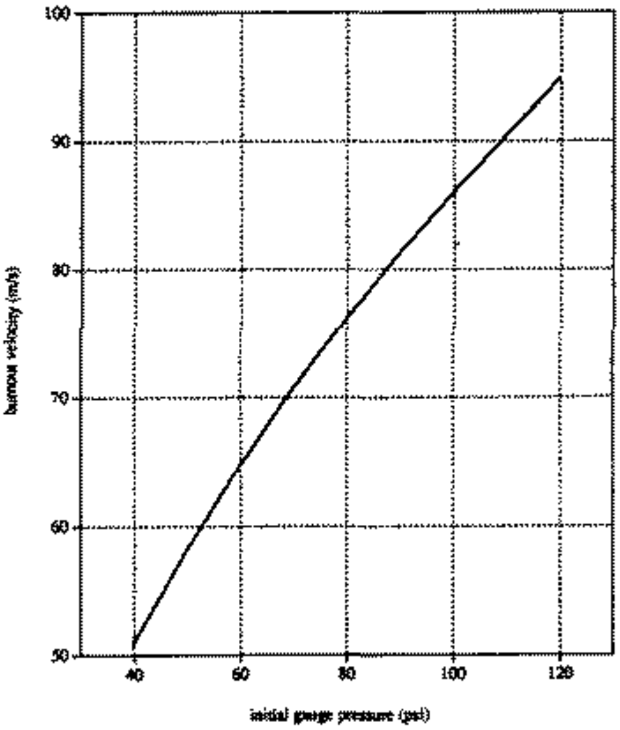

Fig. 8. Burnout velocity vs initial gauge pressure (m = 48.4 g, D_r = 11.0 cm, D_h = 2.15 cm, V_r = 2.08 ℓ, and the initial volume of water is 0.70 ℓ).

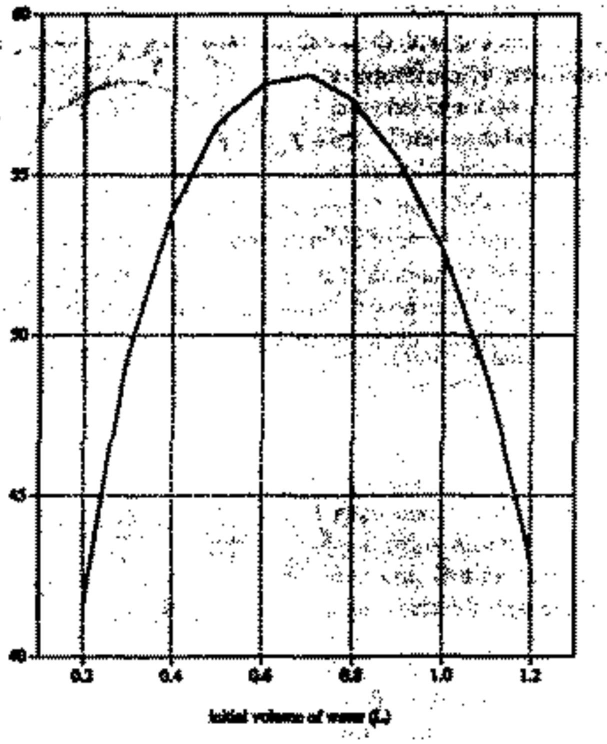

Fig. 9. Burnout velocity vs initial volume of water (m = 48.4 g, D_r = 11.0 cm, D_h = 2.15 cm, V_r = 2.08 ℓ, and the initial gauge pressure is 50 psi).

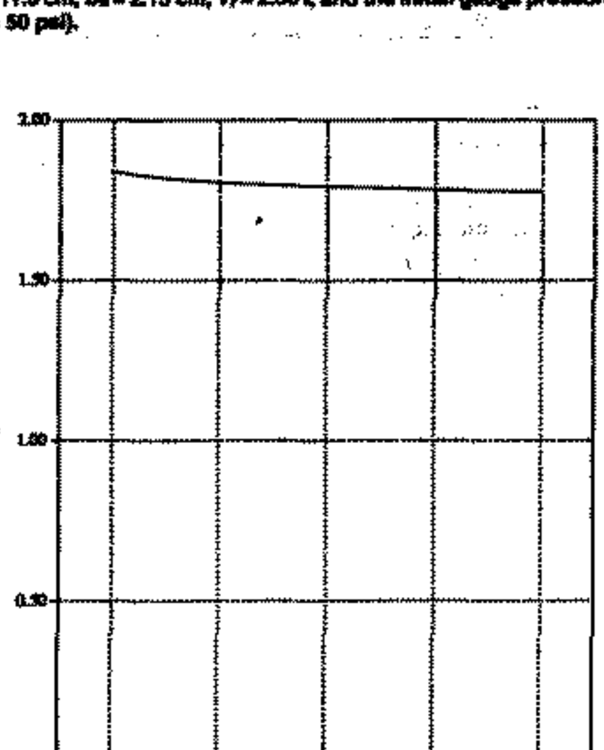

Fig. 10. Burnout height vs initial gauge pressure (m = 48.4 g, D_r = 11.0 cm, D_h = 2.15 cm, V_r = 2.08 ℓ, and the initial volume of water is 0.70 ℓ).

ρ—the density of water (1000 kg/m^3).

V_o —the initial volume of air in the rocket (0–2 ℓ).

P_{atm}—the atmospheric pressure (≈ 14.7 psi).

D_h—the diameter of the hole (≈ 2.15 cm).

D_r —the diameter of the rocket (≈ 11.0 cm).

m_r—the mass of the empty rocket (≈ 0.0484 kg).

V_r—the total volume of the rocket (≈ 2.08 ℓ).

Note our typical values in parentheses.

2. Choose a small increment of the volume of air in the rocket, ΔV, to use for the iterations (0.01→0.03 ℓ).
3. Calculate the value of u for the current value of V using Eq. (3).
4. Calculate $\frac{\Delta V}{\Delta t}$ using Eq. (2) and divide the result into ΔV to get the time increment. Then update the time.
5. Calculate the amount of water remaining in the rocket, V_w, by subtracting the volume of the air, V, from the volume of the rocket, V_r.
6. Calculate the change in the velocity of the rocket using Eq. (1) and use it to update the current value of the velocity.
7. Increment the volume by ΔV and update the current value of the volume, V.
8. Repeat steps 3 through 7 until the volume of air, V, matches the volume of the rocket, V_r.

These types of numerical calculations are most easily performed on spreadsheet programs. Our spreadsheet consists of columns for the volume of air, V, the rate at which this volume changes, $\frac{\Delta V}{\Delta t}$, the time, t, the velocity of the water, u, the velocity of the rocket, v, and the height of the rocket. For an initial volume of water of 0.70 ℓ and an initial gauge pressure of 80 psi, the graphs of the velocity of the rocket as a function of time and height of the rocket vs time are shown in Figs. 6 and 7. The predictions are shocking! When the rocket expels all its water ("burnout") it should reach a speed of 76.3 m/s (171 mph) in a time of only 0.070 s during which *time it will cover 1.80 m. This is an average acceleration of* 111 g's.

We produced curves of burnout velocity vs initial gauge pressure (Fig. 8), burnout velocity vs initial volume of water (Fig. 9), burnout height vs initial gauge pressure (Fig. 10), and burnout height vs initial volume of water (Fig. 11). These curves make many interesting predictions. The highest burnout velocity is achieved with ≈ 0.70 ℓ of water, but it is fairly insensitive to changes of ≈ 0.20 ℓ. Increasing the initial pressure always increases the burnout velocity. The burnout height doesn't depend very much on the pressure, but it does depend almost linearly on the volume.

While this was a wonderful physics problem we had worked out, there was only one way we could believe the results. They needed to be challenged experimentally.

Experimental Results

When we realized that we wanted detailed information about the motion of a macroscopic object during approximately one-tenth of a second, we knew that the only place to turn was the physical education department. After all, they study the intricate and rapid movement of athletes; besides,

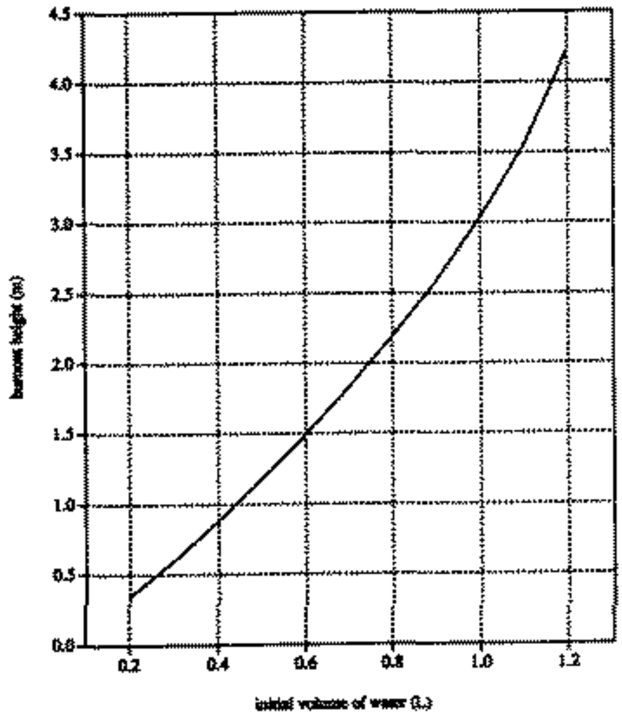

Fig. 11. Burnout height vs initial volume of water (m = 48.4 g, D_r = 11.0 cm, D_h = 2.15 cm, V_r = 2.08 ℓ, and the initial gauge pressure is 50 psi).

they have bigger budgets than we do. We used their high-speed camera[10] to record the launch of the water rocket. The camera was rated at 286 frames per second, so each frame is approximately 3.5 ms apart. Two frames from a launch are shown in Figs. 12 and 13. These frames are roughly 28 ms apart. Notice the graduated board behind the rocket and the clock to the right. The clock is actually a motor turning a disk at 1740 rpm. The board and the clock allowed us to find position and time. The initial volume of water was 0.7 ℓ and the initial gauge pressure was 80 psi.

In Fig. 13 you can see the plume that we suspect is a mixture of water and air that is created as the last of the water is ejected. The order of magnitude of our predictions was right! The burnout height was less than two meters and the time was under one-tenth of a second. Simply incredible!

The position-vs-time data is plotted in Fig. 14. This graph also contains the theory curve and some "time shifted" data. The theory curve and the data curve are not as close as we had hoped. The real rocket actually appears to outperform our theoretical one. We have several possible explanations for the discrepancy. There are problems knowing the precise time of "ignition" because during the first milliseconds the rocket is moving slowly. This would have a tendency to shift the data to the right. To make the data match the theory requires a time shift of about 0.01 s, as shown in Fig. 14. An error of this magnitude means that we misjudged the time of ignition by three or four frames of film, which seems a bit too large. A second experimental problem is that the shaft of the plumbing is about 3 cm inside the bottle, so our model of the motion doesn't begin to apply until the rocket already has an initial velocity that we have neglected. Accounting for this effect would move the theory curve upward by an amount proportional to the elapsed time. Our estimate of the size of this error might explain one-third of the discrepancy. A third potential problem is the assumption of a purely adiabatic expansion of the air inside the rocket. The fact that we see fog in the bottle after launch means that water vapor is being converted into liquid, releasing energy for which we haven't accounted. In addition, there are many other approximations inherent in our theory that could be responsible for the discrepancies.

Many features of our theory were verified by this experiment. For example, we were amazed to see that the water doesn't spray wildly out of the bottle. Instead, it looks as though it is in a pipe. This explains why the burnout height doesn't depend on the initial pressure and it depends linearly on the initial volume of water. The outgoing water essentially creates a cylinder with a diameter equal to the diameter of the hole in the bottle. Varying the volume of water changes the height of this cylinder in a linear fashion. Varying the pressure changes the rate at which the cylinder of water is created, but not its total volume.

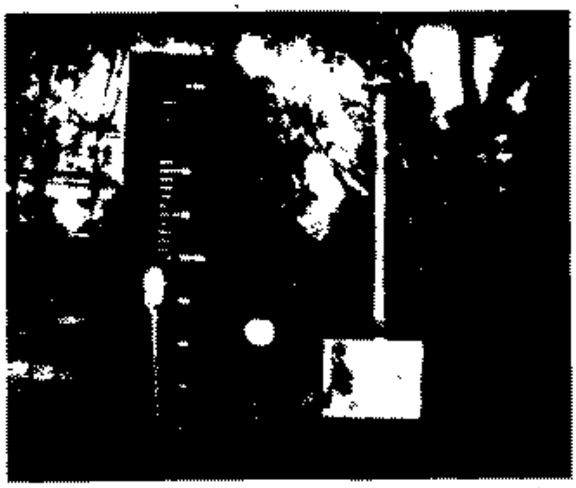

Fig. 12. High-speed photograph approximately 55 ms into the launch. Notice the column of ejected water.

Fig. 13. High-speed photograph approximately 80 ms into the launch. Note the plume of water and air created at burnout.

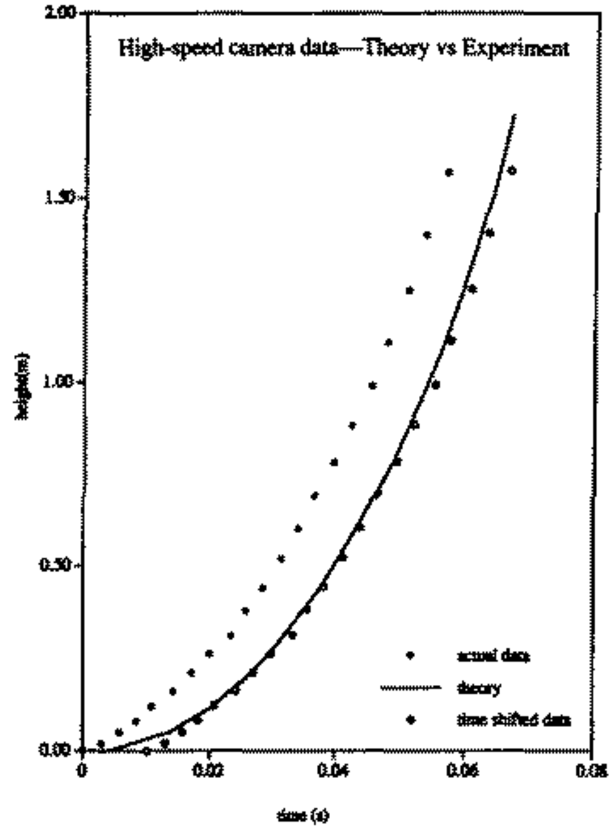

Fig. 14. Position-vs-time data from the high-speed camera plotted along with the theoretical prediction. The time shifted data has been moved 0.01 s to the right.

Another amazing feature of these photographs is that the water is indeed gone after the first two meters, yet when you look upward after a launch, you will see water falling from heights of five to ten meters. The water that is ejected by the rocket is, after a few milliseconds, actually traveling upward with respect to the Earth while it is moving backwards relative to the rocket.

The film[11] for this high-speed camera is fairly expensive and it takes a long time to develop because the company[12] waits to receive several rolls of this type of film before processing it all as a single batch. The film and developing costs about $40 per 100 ft; in our first 100 ft we got only one good recording of a launch. These problems with the photographic technique, as well as the discrepancies between theory and experiment, led us to seek a different method of data collection for more systematic studies.

About this time we came upon the *TPT* article by Randy Jenkins[13] in which he described the use of PASCO's Smart Pulley to collect data from solid fuel rockets. Adopting his technique, we tied a light string (approximately 2.5 m long) to the rocket. We drilled a small hole from the inside of the channel in the pulley to the outside, slid the other end of the string through the hole, held it in place with masking tape, and rolled up the remaining string onto the pulley. The Smart Pulley was connected directly to a computer. When we launched the rocket, it unrolled the string from the pulley and easily removed the masking tape as it left. The data was collected in the computer and analyzed. The string seemed to have almost no effect on the flights of the rockets, unless you count the fact that several rockets now hang like giant Christmas-tree bulbs on nearby sycamore trees. If you intend to try this method of data collection, be aware that you must keep water away from the pulley's infrared detector. You might also want to keep the computers out of the back spray from the rockets. Figure 15 shows several members of our team preparing for a launch.

We had hoped to complete two sets of studies: constant initial volume of water (0.7 ℓ) as the initial gauge pressure was varied from 40 to 120 psi and constant gauge pressure (50 psi) as the initial volume of water was varied from 0.20 to 1.20 ℓ. We had hoped to generate the data necessary to test the theoretical curves of Figs. 8–11.

Figure 16 shows the height-vs-time data, along with the predictions of the theory for three launches at 50 psi and 0.70 ℓ and three launches at 100 psi and 0.70 ℓ. Notice that the three launches at 50 psi were very consistent with each other, but still seemed to suffer from over performance with respect to the theory. The launches at 100 psi showed a serious lack of consistency, but still outperformed the theory in all three cases. Part of the problem here is that our launcher gets stubborn at these high pressures, which means that the pressurized rocket sits on the pad a long time before launching.

Fig. 15. The WRECKS (Water Rocket Experimental Center for Kinematic Studies) team in action. One of the authors (Klein) is at the bottom left checking the launch device.

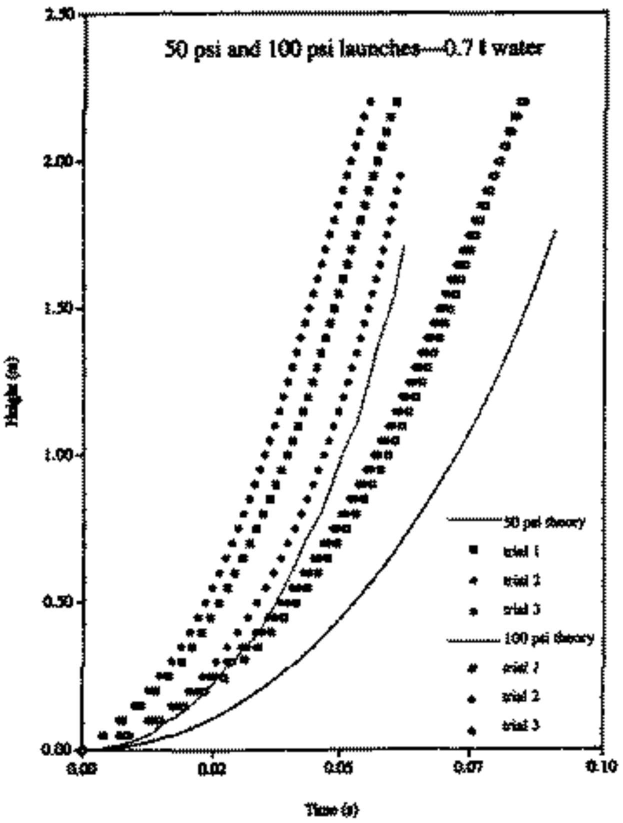

Fig. 16. Several launches at two different pressures. Data from the Smart Pulley setup plotted along with the theory.

During this time water gets driven back into the plumbing assembly, reducing both the initial volume of water and the air pressure. Part of the discrepancy between theory and experiment can be explained by the fact that we were using a three-hole Smart Pulley. So once again, by the time we started getting data the rocket had an initial velocity that isn't accounted for in our model. On the other hand, at this point it may be time to seriously consider the possibility that the discrepancy is a real physical effect.

Summary

About this time we came to the conclusion that we needed more and better equipment. We are sad to report that due to the end of the Cold War interest in the behavior of rockets doesn't seem to be a priority with the usual funding agencies. We were forced to bring our project to an end (besides the end of the school year was fast approaching and it was about time that we got to work on other things).

In summary, we have shown how to build a fun and exciting physics toy that will amaze and delight even the most jaded physics student or teacher. It is really a great collection of fortuitous circumstances that even allows the 2-ℓ soda-bottle water rocket to be practical. The bottles are disposables and therefore cheap. Half-inch PVC pipe is inexpensive, available, and it fits perfectly. The bottles are designed to withstand the needed pressure. There is even a plastic ring on the bottle for attaching the launch mechanism. In addition, the hole in the bottle is just the right size for the necessary fluid dynamics. (We tried to make it smaller, but the water rocket became very unstable and dangerous.)

Our theory is based on a lovely physics problem that *illustrates many of the fundamental concepts treated in an* introductory course. Additionally, it points out the need for students to be able to use numerical methods, such as spreadsheets, if they want to be able to solve "real-world" physics problems. The predictions of our theory, while not in complete agreement with experimental results, are within 10 to 20%. Considering the unbelievable numbers the theory predicts and the uncertainties in our measurements, we are quite satisfied with a discrepancy of this magnitude.

There is a whole array of questions that our work has left unanswered: What is the source of the discrepancy? Can the timing be improved? Can the experiment be redesigned so that knowing when the rocket starts moving is not so critical? Should the energy released by the condensation of water be included in the theory? If so, how? Why is the size of the hole in the bottle so crucial? We leave these questions for you to investigate. Our study of the launch of a 2-ℓ soda-bottle water rocket has made a big splash on our campus; we hope that it will lead you to new heights.

Acknowledgments

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References

1. W. Esler and D. Sanford, *Sci. Teach.* **56** (5), 20 (1989).
2. W.P. Palmer, *Aust. Sci. Teach. J.* **37** (1), 34 (1991).
3. According to Ref. 2, a launch kit is marketed in Australia under the name "ROKIT." It is available from Omega Scientific, 34 Columbia Cr., Modbury North, SA 5092; phone 08-273-34119. These same kits appear to be available from Edmund Scientific, *Department 15B1, E910 Edscorp Building, Barrington*, NJ 08007; phone: 609-573-6886; catalog # M39,978.
4. We chose these horrible units because our gauge was calibrated in psi. In our calculations the pressure is always used in ratio with the initial pressure, so these units don't cause a problem.
5. For a more complete derivation of the rocket equation see Raymond Serway, *Physics for Scientists and Engineers*, 3rd ed. (Saunders, New York, 1990), pp. 232–233.
6. Ibid., pp. 405–406.
7. Ibid., p. 404.
8. Ibid., pp. 568–569.
9. Ibid., p. 566.
10. The camera was a 16-mm-1B/1BAC Rotary Prism Camera manufactured by Photo-Sonics Inc., Burbank, CA.
11. The film was #7278 Eastman Tri-X Reversal. We ordered it directly from Eastman Kodak, Hollywood, CA. The cost was approximately $18 for 100 ft.
12. The film was developed by Alpha-Cine Lab, Seattle, WA. The cost was approximately $21 for 100 ft.
13. R.A. Jenkins, *Phys. Teach.* **31**, 10–15 (1993).